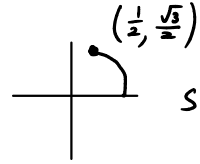
MATH 119: Midterm 2

Name: Key

Directions:

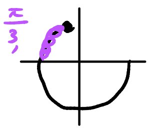
* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
		60



$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

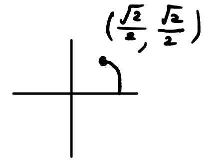
1. Simplify these expressions:



$\cos\left(-\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right)$

$= -\frac{1}{2}$

* $\sin^2\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{-4\pi}{3}\right) + 3\tan\left(\frac{\pi}{4}\right)$



$\tan\left(\frac{\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + 2 \cdot \left(-\frac{1}{2}\right) + 3 \cdot 1$

LoE #5

$= \frac{(\sqrt{3})^2}{2^2} - \frac{2}{2} + 3$

frac law #1

$= \frac{3}{4} - 1 + 3 = \frac{3}{4} + 2$

$= \frac{3}{4} + \frac{8}{4}$

$= \frac{3+8}{4}$

$= \boxed{\frac{11}{4}}$

* $\underbrace{\frac{1}{1+\sin\theta}} + \frac{1}{1-\sin\theta}$

Form: frac + frac

use frac law #3 : common denominator.

$\frac{1-\sin\theta}{1-\sin\theta} \cdot \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta} = \frac{1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} + \frac{1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$

$(A-B)\cdot(A+B) = A^2 - B^2$

$(1-\sin\theta)(1+\sin\theta) = 1-\sin^2\theta$

$= \frac{1-\sin\theta + 1 + \sin\theta}{1-\sin^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\downarrow$

$\cos^2\theta = 1 - \sin^2\theta$

$= \frac{2}{\cos^2\theta}$

5.2 identity

$= \boxed{2\sec^2\theta}$

2. Short answer questions.

   ⚠ Justify each answer with formulas or facts for full credit; do not just write "yes" or "no" ⚠ .

   (a) Given $f(x) = \sin(x)$, does there exist $x \in \mathbb{R}$ such that $f(x) = 2$? Why or why not?

   No, the maximum y-coordinate on the unit circle is 1, no matter where you walk to.

   (b) If a mass attached to a spring is moving in simple harmonic motion, can we use the function

   $$d(t) = a\tan(\omega t)$$

   to model it's displacement? Why or why not?

   No, simple harmonic motion is modeled by $a\sin(\omega t)$ or $a\cos(\omega t)$.

   tangent is inappropriate because $\tan(x) \to \infty$ as $x \to \frac{\pi}{2}$ from the left

   so a spring would have to stretch infinitely.

   (c) Is it possible for linear speed to be less than angular speed? Why or why not?

   Yes, angular speed is $\omega = \frac{\theta}{t}$

   linear speed is $v = \frac{s}{t} = \frac{r\theta}{t} = r \cdot \frac{\theta}{t} = r \cdot \omega$

   If $r < 1$ then linear speed is less than angular speed.

   (d) When proving a trig identity, are we allowed to square both sides? Why or why not?

   No, you need to either

   ① start with one side, perform steps to reach the other or

   ② simplify both sides and "meet in the middle"

3. Prove these identities:

* $\dfrac{1}{\sin x} - \sin x = \cot x \cdot \cos x$

$$LHS = \frac{1}{\sin x} - \sin x \cdot \frac{\sin x}{\sin x} = \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$\sin^2 x + \cos^2 x = 1$

$\downarrow$

$\cos^2 x = 1 - \sin^2 x$

$$= \frac{\cos^2 x}{\sin x}$$

frac law #1

$$= \frac{\cos x}{\sin x} \cdot \cos x \overset{5.2}{=} \cot x \cdot \cos x = RHS \quad \blacksquare$$

$1 - \sin^2\alpha - (1 - \cos^2\beta)$

* $\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta$

Start w/ LHS; looks like addition identity.

$$LHS = \cos(\alpha+\beta)\cos(\alpha-\beta) \overset{7.2}{=} (\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$(A - B)\cdot(A + B)$$

$$\overset{A^2-B^2}{=} \cos^2\alpha\cos^2\beta - \underbrace{\sin^2\alpha}\,\underbrace{\sin^2\beta}$$

convert to it's $1-\cos^2$ version

$$= \cos^2\alpha\cos^2\beta - (1-\cos^2\alpha)(1-\cos^2\beta)$$

expand dist law

$$= \cos^2\alpha\cos^2\beta - \left[(1-\cos^2\alpha)\cdot 1 - (1-\cos^2\alpha)\cdot\cos^2\beta\right]$$

$$= \cos^2\alpha\cos^2\beta - \left(1 - \cos^2\alpha - \cos^2\beta + \cos^2\alpha\cos^2\beta\right)$$

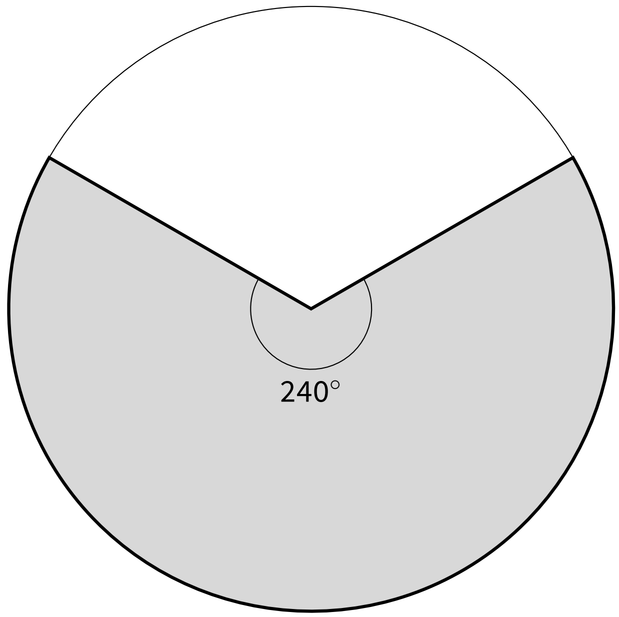
$$= \cancel{\cos^2\alpha\cos^2\beta} - 1 + \cos^2\alpha + \cos^2\beta \cancel{- \cos^2\alpha\cos^2\beta}$$

factor out -1

$$= \cos^2\alpha - (1 - \cos^2\beta)$$

$$\overset{5.2}{=} \cos^2\alpha - \sin^2\beta = RHS \quad \blacksquare$$

4. Suppose the shaded region is 6π in^2. Find the radius of the circle; your answer should be an integer.



area of sector:

$$A = \frac{1}{2} r^2 \theta \quad , \quad \theta \text{ in rad.}$$

$$6\pi = \frac{1}{2} r^2 \cdot \overset{4}{\cancel{240}} \cdot \frac{\pi}{\underset{3}{\cancel{180}}}$$

$$6\pi = \frac{\overset{2}{\cancel{4}}\,\pi}{\cancel{2} \cdot 3} \cdot r^2$$

$$\frac{3}{\cancel{2\pi}} \cdot \overset{3}{\cancel{6\pi}} = \cancel{\frac{2\pi}{3}} \cdot r^2 \cdot \frac{3}{\cancel{2\pi}}$$

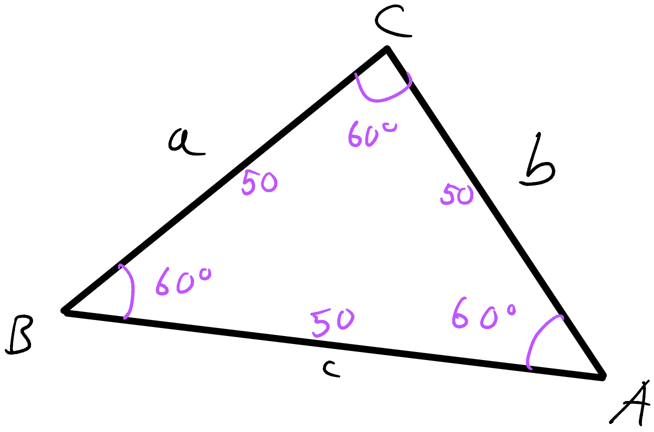
$$9 = r^2$$

$\sqrt{r^2} = \pm\sqrt{9}$ ← + version as r is not negative

$$r = \sqrt{9} = 3$$

$$\boxed{r = 3}$$

5. Suppose a triangle has $a = 50, b = 50, \angle A = 60^\circ$. Solve the triangle.



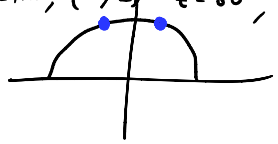
Solve for $\angle B$

$$\cancel{50} \cdot \frac{\sin 60^\circ}{\cancel{50}} = \frac{\sin B}{\cancel{50}} \cdot \cancel{50}$$

$$\sin B = \sin 60^\circ$$

$$\sin B = \frac{\sqrt{3}}{2}$$

$t = 120^\circ, \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $\quad t = 60^\circ, \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



So $\angle B = 60^\circ$ or $\angle B = 120^\circ$, possibly two solution case. But if $\angle B$ were 120°,

then $180^\circ = \angle A + \angle B + \angle C$

$$180^\circ = 60^\circ + 120^\circ + \angle C$$

$\angle C = 0^\circ$ impossible!

So one solution case.

$$\boxed{\angle B = 60^\circ}$$

Solve for $\angle C$:

$$180^\circ = \angle A + \angle B + \angle C$$

$$180^\circ = 60^\circ + 60^\circ + \angle C$$

$$\boxed{\angle C = 60^\circ}$$

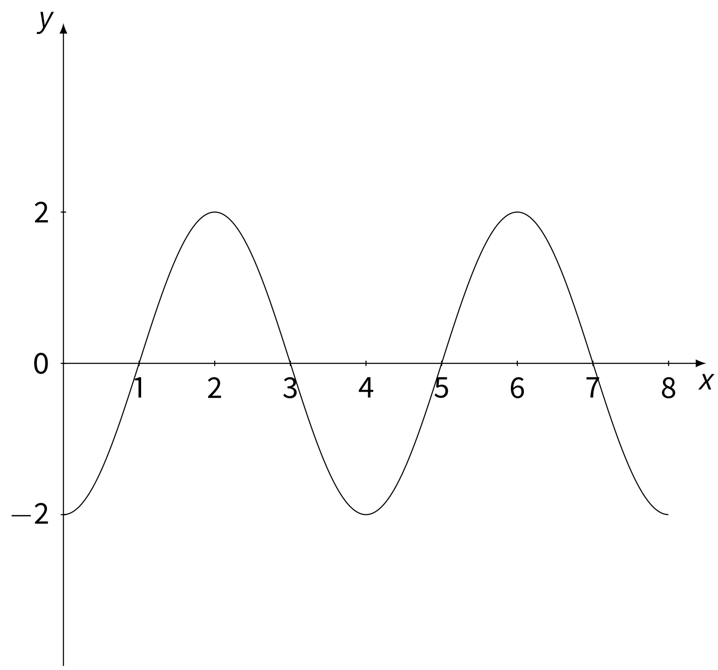
Solve for side c:

$$\frac{\sin 60^\circ}{50} = \frac{\sin 60^\circ}{c}$$

$$c = \cancel{\sin 60^\circ} \cdot \frac{50}{\cancel{\sin 60^\circ}}$$

$$\boxed{c = 50}$$

6. Suppose a mass attached to a spring is moving in simple harmonic motion. The displacement $f(t)$ is shown in the following graph.



Here, t is measured in seconds and $f(t)$ is measured in centimeters.

(a) Find a function $f(t)$ describing the displacement.

$y = a\cos\omega t$ since cos starts at 1, we just transform it.

$\boxed{a = 2}$. period is 4. solve for ω. so $\frac{2\pi}{\omega} = 4 \longrightarrow \omega = \frac{2\pi}{4} = \frac{\pi}{2}$.

cosine is reflected around x-axis.

$$\boxed{y = -2\cos\left(\frac{\pi}{2}t\right)}$$

(b) How many centimeters is the mass displaced at time $t = \frac{3}{2}$?

$$y = -2\cos\left(\frac{\pi}{2}\cdot\frac{3}{2}\right)$$

$$= -2\cos\left(\frac{3\pi}{4}\right)$$

$$= -2\left(-\cos\left(\frac{\pi}{4}\right)\right)$$

$$= 2\cdot\frac{\sqrt{2}}{2} = \boxed{\sqrt{2}\text{ cm}}$$